Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

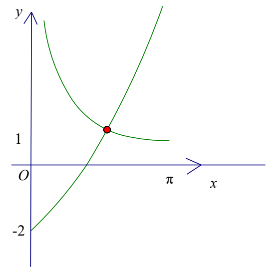
Question	Answer	Marks	Guidance
1	State correct unsimplified first two terms of the expansion of $(1-2x)^{\frac{1}{2}}$, e.g. $1+\frac{1}{2}(-2x)$	**B1**	Symbolic coefficients are not sufficient. $1-x$
	State correct unsimplified term in x^2, e.g. $\dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)(-2x)^2}{2!}$	**B1**	Symbolic coefficients are not sufficient. $-\frac{1}{2}x^2$
	Obtain sufficient terms of the product of $(3 + x)$ and the expansion up to the term in x^2	**M1**	
	Obtain final answer $3 - 2x - \frac{5}{2}x^2$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	Use law of logarithm of a product (or quotient) on correct terms	***M1**	
	Use correct method to eliminate logarithm	**DM1**	
	Obtain a correct quadratic in x, e.g. $x^2-5x-\mathrm{e}^7=0$ (allow decimals)	**A1**	
	Obtain answer $x = 35.71$ only	**A1**	
		4	

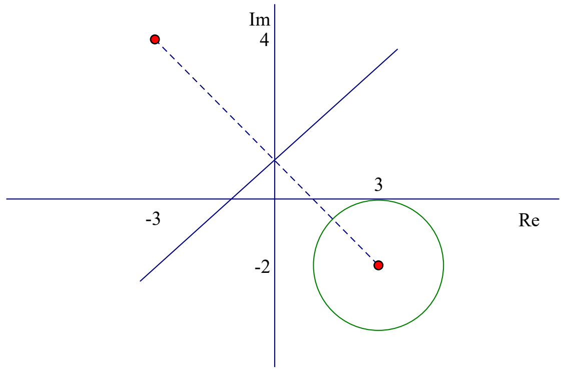
Question	Answer	Marks	Guidance
3	State or imply that $y\ln a = \ln b + \ln x$	B1	
	Carry out a completely correct method for finding $\ln a$ or $\ln b$	M1	E.g., from $\ln a = \ln b + 0.336$ $1.5\ln a = \ln b + 1.31$.
	Obtain value $a = 7$	A1	
	Obtain value $b = 5$	A1	
		4	

Question	Answer	Marks	Guidance
4(a)	State or imply $r = 2$	B1	
	State or imply $\theta = -\frac{2}{3}\pi$	B1	
		2	
4(b)	State or imply $r = \frac{5}{2}$	B1FT	FT $\dfrac{5}{\textit{their}\ 2}$.
	State or imply $\theta = \frac{5}{6}\pi$	B1FT	FT $\frac{1}{6}\pi - \textit{their} -\frac{2}{3}\pi$.
		2	

Question	Answer	Marks	Guidance
5	Use correct quotient (or product) rule	***M1**	
	Obtain correct derivative $\dfrac{e^{\sin x}\cos^3 x - (-2e^{\sin x}\sin x\cos x)}{\left(\cos^2 x\right)^2}$ or equivalent	**A1**	
	Equate numerator to zero	**DM1**	
	Obtain equation in one unknown	**DM1**	E.g. $\sin^2 x - 2\sin x - 1 = 0$.
	Solve a 3 term quadratic in $\sin x$ to find a value for x	**M1**	
	Obtain a correct solution to the quadratic equation, e.g. 3.57°	**A1**	At least 3sf.
	Obtain a further correct solution, e.g. $x = 5.86°$ and no others in the interval	**A1FT**	At least 3sf. FT 3π – *their* 3.57.
	Alternative Method for the first 3 marks:		
	Take logarithms of both sides and simplify	**(*M1)**	$\ln y = \sin x - 2\ln\cos x$ or equivalent.
	Obtain $\dfrac{1}{y}\dfrac{dy}{dx} = \cos x + 2\dfrac{\sin x}{\cos x}$	**(A1)**	Or equivalent.
	Equate $\dfrac{dy}{dx}$ to zero	**(DM1)**	
	Continue as for the original		
		7	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ Correct shape, correct vertical intercept	B1	
	Sketch a second relevant graph, e.g. $y = \operatorname{cosec}\frac{x}{2}$ (correct shape, minimum above the axis) and justify the given statement. Need to mark intersection with a dot, a cross, or say root at points of intersection, or equivalent	B1	
		2	
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	M1	Use of degrees is M0.
	Complete the argument correctly with correct calculated values	A1	E.g. $-0.282 < 2.086$, $4.389 > 1.188$ $1 < 1.626$, $2 > 1.432$ $2.36 > 0$, $-3.2 < 0$ At least 2sf. Condone truncation.
		2	
6(c)	State $x = \ln\left(\operatorname{cosec}\frac{1}{2}x + 3\right)$ and rearrange to the given equation $\operatorname{cosec}\frac{x}{2} = e^x - 3$	B1	AG. Or vice versa and obtain the iterative formula.
		1	
6(d)	Use the iterative formula correctly at least twice	M1	Use of degrees in M0 (might see 1.38….).
	Obtain final answer 1.50	A1	
	Show sufficient iterations to 4 dp to justify 1.50 to 4 dp or show there is a sign change in the interval (1.495, 1.505)	A1	1.5156, 1.4940, 1.4978, 1.4971.
		3	

Question	Answer	Marks	Guidance
6(e)	4	**B1**	
		1	

Question	Answer	Marks	Guidance
7(a)	Show a circle centre (3, –2)	**B1**	
	Show a circle with radius 2 FT centre not at the origin	**B1FT**	
	Show the point representing (–3, 4) or the midpoint (0, 1)	**B1**	
	Show the perpendicular bisector of the line joining (–3, 4) and centre of the circle FT is on the position of (–3, 4) and centre of the circle	**B1FT**	
		4	
7(b)	Carry out a correct method for finding the least value of $\|z-w\|$	**M1**	(Distance (3, –2) to (0, 1)) – 2.
	Obtain answer $\sqrt{18}-2$ or $3\sqrt{2}-2$	**A1**	
		2	

Question	Answer	Marks	Guidance
8	State or imply du = – cos$x$ dx	**B1**	
	Use sin 2x = 2sin$x$cos$x$ and write the integral in terms of u	***M1**	
	Obtain $\pm 2\int \frac{(1-u)}{\sqrt{u}}\,du$ or equivalent	**A1**	
	Integrate correctly to obtain $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$	**DM1**	
	Obtain correct $-4u^{\frac{1}{2}} + \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
	Correctly use limits u = 2 and 0 in an expression of the form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$ OR limits $x = \frac{3}{2}\pi$ and $\frac{1}{2}\pi$ in an expression of the form $a(1-\sin x)^{\frac{1}{2}} + b(1-\sin x)^{\frac{3}{2}}$	**DM1**	
	Obtain $\frac{8}{3} - \frac{4}{3}\sqrt{2}$	**A1**	
		7	

Question	Answer	Marks	Guidance
9(a)	Carry out correct process for evaluating the scalar product of direction vectors, equate the result to zero and obtain given value of $a = 4$	**B1**	E.g. $2(3) + (-1)(-2) + a(-2) = 0$.
		1	
9(b)	Express general point of at least one line correctly in component form, i.e. $(1 + 2\lambda, -2 - \lambda, 3 + 4\lambda)$ or $(-1 + 3\mu, -1 - 2\mu, -1 - 2\mu)$	**B1**	The third component could be implied by a correct final answer.
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain $\lambda = -1$ or $\mu = 0$	**A1**	
	Obtain position vector of point of intersection is $-\mathbf{i} - \mathbf{j} - \mathbf{k}$	**A1**	
		4	
9(c)	Equate one component of l_1 to matching component of A and solve to find λ	**M1**	
	Use $\lambda = -3$ in equation of l_1 and show this gives position vector of A	**A1**	AG Or show $\lambda = -3$ for all three components equated.
		2	
9(d)	Method to find position vector of B	**M1**	E.g. $\pm$ 2×*their* $(-\mathbf{i} - \mathbf{j} - \mathbf{k}) \pm (-5\mathbf{i} + \mathbf{j} - 9\mathbf{k})$
	Obtain position vector of B is $3\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$	**A1**	
		2	

Question	Answer	Marks	Guidance
10(a)	Obtain $2 = \sec^2 y \dfrac{dy}{dx}$ or equivalent	B1	E.g. $2\dfrac{dx}{dy} = \sec^2 y$ by differentiation with respect to y.
	Use $\sec^2 y = 1 + \tan^2 y$	M1	
	Replace $\tan y$ with $2x$ and rearrange to obtain given answer $\dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$	A1	
		3	
10(b)	Integrate by parts and reach $ax^2 \tan^{-1} 2x + b\int \dfrac{x^2}{1+4x^2}\,dx$	*M1	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \int \dfrac{x^2}{1+4x^2}\,dx$	A1	OE
	Reduce integral to expression of the form $\int m + \dfrac{n}{1+4x^2}\,dx$	M1	
	Complete integration and reach $px^2 \tan^{-1} 2x + qx + r\tan^{-1} 2x$	M1	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \frac{1}{4}x + \frac{1}{8}\tan^{-1} 2x$	A1	OE
	Use limits of $x = \frac{1}{2}$ and $x = \frac{1}{2}\sqrt{3}$ in the correct order, having integrated twice	DM1	
	Obtain answer $\frac{5}{48}\pi - \frac{1}{8}\sqrt{3} + \frac{1}{8}$ or exact equivalent	A1	
		7	

Question	Answer	Marks	Guidance
11(a)	State or imply equation of the form $\frac{dx}{dt} = kx(300 - x)$ and use $\frac{dx}{dt} = 0.2$ and $x = 1$	M1	M0 for verification.
	Obtain $k = \frac{1}{1495}$ and rearrange to the given answer	A1	$1495\frac{dx}{dt} = x(300 - x)$.
		2	

Question	Answer	Marks	Guidance
11(b)	Separate variables correctly	B1	$\int \frac{1}{x(300-x)}\mathrm{d}x = \int \frac{1}{1495}\mathrm{d}t$
	Correct integration of t term	B1	E.g. obtain t or $\frac{t}{1495}$.
	State or imply partial fractions of the form $\frac{A}{x}+\frac{B}{300-x}$	B1	
	Correct method to find A or B	M1	$A=\frac{1}{300}$ and $B=\frac{1}{300}$. May see $A=B=\frac{1495}{300}=\frac{299}{60}$.
	Obtain terms $\frac{1495}{300}\ln x-\frac{1495}{300}\ln(300-x)$	A1	OE. May see $\frac{1}{300}\ln x-\frac{1}{300}\ln(300-x)$.
	Use $t=0$, $x=1$ to evaluate a constant or as limits in a solution containing terms of the form $\ln x$, $\ln(300-x)$ and t.	M1	
	Obtain correct answer in any form	A1	E.g. $\frac{1495}{300}[\ln x-\ln(300-x)]=t-\frac{1495}{300}\ln 299$.
	Use law of logarithms twice to obtain an expression for t	M1	
	Obtain final answer $t=\frac{299}{60}\ln\frac{299x}{300-x}$ or equivalent single logarithm	A1	
		9	